REFLECTIONS ON AMPLITUDES*

BY

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Abstract

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Modern seismic recording instruments allow precise measurements of the amplitude of reflected signals. Intuitively we would expect that this amplitude information could be used to increase our knowledge of the physical properties of the reflecting earth.

The relevant factors defining the amplitude of a reflection signal are: spherical divergence, absorption, the reflection coefficient of the reflecting interface, the cumulative transmission loss at all interfaces above this, and the effect of multiple reflections.

Of these factors, three—spherical divergence, the reflection coefficient and the transmission loss—are reasonably clear concepts (though the estimation of transmission loss from acoustic logs caused some difficulties in the hey-day of synthetic seismograms). Absorption still presents considerable problems of detail, but our understanding has increased significantly in recent years.

The factor least well understood is undoubtedly the effect of multiple reflections. Multiple paths having an even number of bounces can have the effect of delaying, shaping and magnifying the pulse transmitted through a layered sequence. Simple demonstations of this phenomenon can be made using elementary thin plates, and these can be presented for various synthetic and real sequences of layers. Such demonstrations lead one to explore the relation between the spectrum of the transmitted pulse and the spectrum of the reflection coefficient series.

If it were possible to isolate the amplitude and shape variations imposed by absorption within a layer, there would be a chance that this measure of absorption would be useful as a correlatable or diagnostic indication of rock properties. If it were possible to isolate the amplitude and shape variations imposed by multiple reflections, there would be a chance that this measure would be useful as an indication of cyclic sedimentation and of the dominant durations of the sedimentary cycles. However, the separation of these two effects constitutes a formidable challenge. The very difficulty of this separation suggests that it may be opportune to review the quantitative estimates of absorption made by field experiments.

Introduction

Why is it that in some areas we need 100 kg of explosive, while in others we need little more than a cap?

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Part of the answer lies, of course, in the noise background. But there must be more to it than that; in the early part of the record, long before the amplitude of the reflection signal dies away to the noise level, we observe one amplitude decay rate in one area and a grossly different one in another. Why?

And why is it that no routine quantitative use is made of seismic amplitudes? Surely the amplitudes must be related to the geology in some meaningful way?

Indeed, was not this one of the principal considerations which led us to adopt binary-gain recording with such enthusiasm? What happened?

We cannot answer these questions fully. Nevertheless it seems opportune to study seismic amplitudes in some detail, and to take note of any features of the amplitude decay which might possibly be indicative of the geology.

We start with a review of the factors which determine the amplitude variations of the seismic reflection signal.

Factors Affecting Reflection Amplitudes

In this study we are concerned primarily with the variations in reflection amplitudes imposed by the subsurface geology. Thus we exclude amplitude considerations which merely define the *scale*—such factors as instrument sensitivity, source energy, and the geophone-ground coupling—and we assume a broad-band instrumental response. This allows us to define five major factors affecting the variations of amplitude: spherical divergence, interface reflection coefficients, absorption, interface transmission losses, and multiple reflection effects. These we discuss in turn.

1. *Spherical divergence*

The familiar law of conservation of energy, when applied to a spherical wavefront emanating from a point source in a uniform lossless material, tells us that the intensity diminishes as the inverse square of the radius of the wavefront (figure 1*a*). Translated into the type of measurements made in seismic work, this says that the pressure amplitude of the seismic wave is inversely proportional to the distance travelled. As always, we are grateful when nature produces a simple relationship.

But nature is just mocking us. The earth is not uniform, and in the presence of an increase of seismic velocity with depth the wavefronts are generally not spherical. Therefore the amplitude decay is subject to an additional effect associated with refraction (figure 1*b*).

For a representative case, the decay of amplitude due to spherical divergence is illustrated in figure 2. The overall decay of 50 dB is referred to a very early reflection at 0.1 s, and thus is appropriate only to a geophone close to the source; more distant geophones would record much less decay. In both cases we may well find that spherical divergence accounts for the majority of the

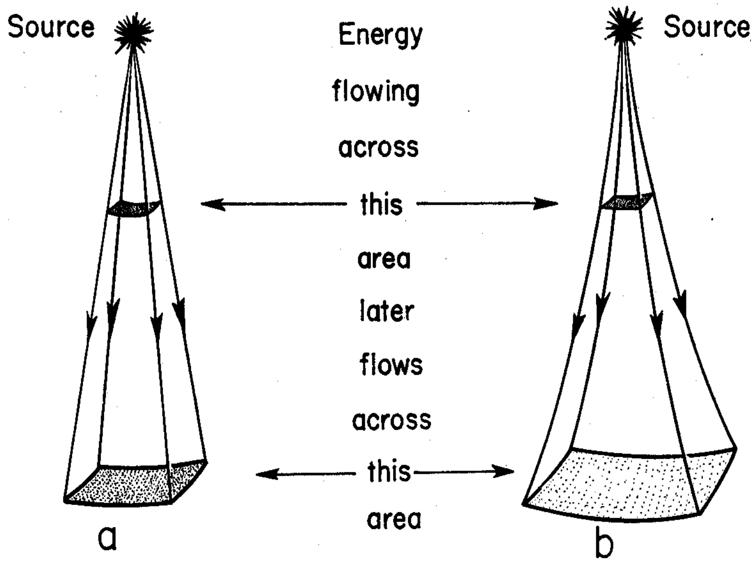


Fig. 1. The inevitable decay in amplitude associated with geometrical divergence (*a*) in a uniform material (*b*) in a material whose velocity increases with depth.

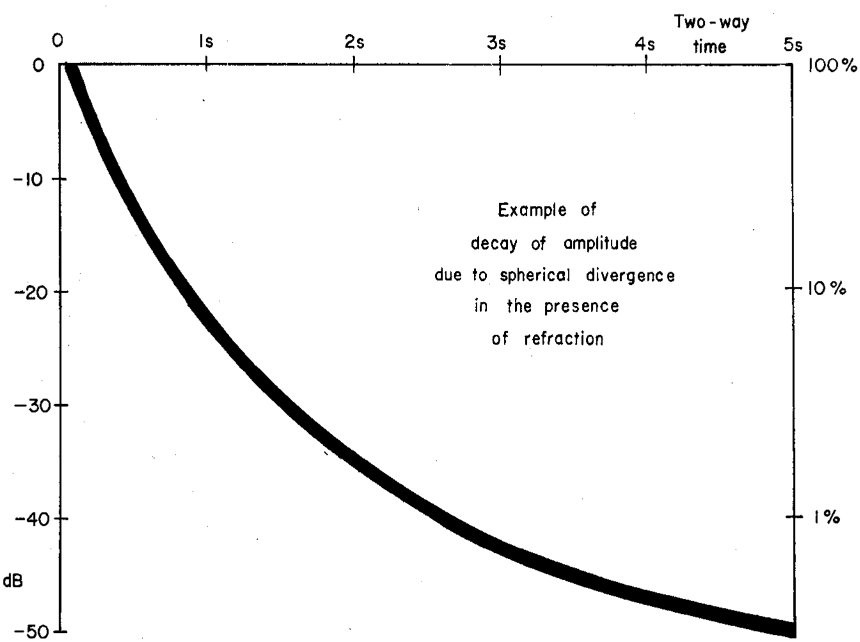


Fig. 2. The magnitude of the geometrical decay, in a typical case. The 0 dB level represents the amplitude of a supposed "first" reflection at 0.1s.

total decay observed on our records; we shall see later what are the circumstances under which this is so.

Spherical divergence in itself conveys no geological information, and so we hasten to compensate it. In the past, this has often been done by multiplying each sample by a factor proportional to the depth (or even the raw time) to which it corresponds; in the following material, however, we are assuming that the compensation also takes full account of refraction.

Even so, the compensation cannot be all we would wish. For example:

—In the usual case when primary reflections and long-period multiples may arrive simultaneously, but with different effective velocities, it is not possible to provide exact compensation for both. Proper compensation for the primaries ordinarily leaves the multiples too large.

—Similarly, compensation on the basis of horizontal layering ordinarily yields an excess of amplitude for reflectors showing strong dip.

—The law assumes a point source. Much seismic work nowadays is done with a source array; such arrays appear as a point source at low frequencies, but may be appreciably directional (showing a less-than-spherical loss) at higher frequencies. Properly, therefore, divergence compensation for such sources should include a frequency-dependent term.

The complications represented by these three items should not worry us too much. Basically, spherical divergence is an effect which is highly predictable and simply understood.

2. *Interface reflection coefficients*

As we know, a seismic reflection is generated at every geological interface across which there is a contrast of acoustic impedance. For present purposes, the acoustic impedance is represented by the product of density and velocity, or ρV. Then at normal incidence (and we must note this limitation) the pressure-amplitude reflection coefficient is given by the difference of the ρV values divided by the sum of the ρV values.

The reflection coefficient, we remember, is not a measure of the physical or geological properties of a layer, but only of the contrast of properties between two layers.

Within most sedimentary sequences, a reflection coefficient of $\pm$ 0.2 would be regarded as large. Values higher than this are observed, but (except near the surface) these values are unusual. Values of $\pm$ 0.1 are found in abundance, and lower values in profusion.

For seismic purposes, the geologic column is represented by a *reflection coefficient series* (or reflection coefficient log), identifying and quantifying the interface contrasts (figure 3). This is, of course, the conceptual germ of the synthetic seismogram.

If it were possible to isolate from the seismic reflection record the reflection coefficient series (with all the magnitudes correct), and if by an independent measurement we could establish the product of density and velocity in the first two layers, then in principle the acoustic impedance in every other layer could be computed. If interval velocities are known by other means, then layer densities could be deduced.

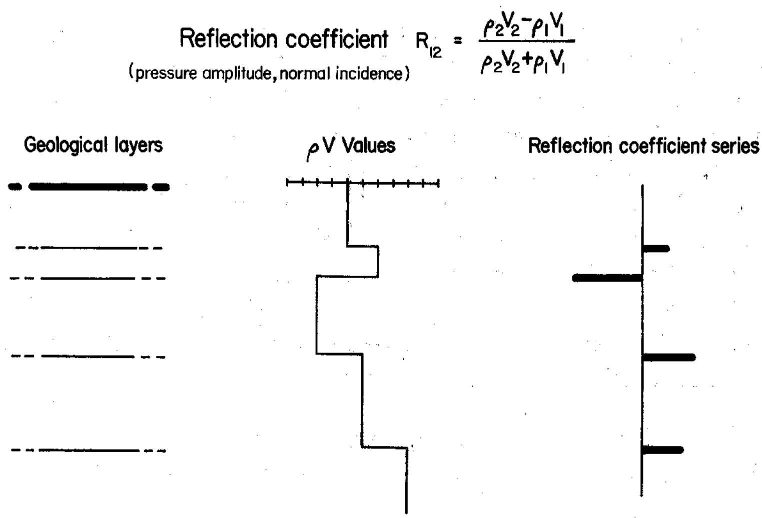


Fig. 3. The physical reality of a layer sequence (left) may be depicted in terms of its ρV log (centre) or its corresponding reflection coefficient series (right).

Therefore one of the long-term objectives of the seismic method must be the determination of the reflection coefficient series with all its magnitudes correct. The determination of the detailed *shape* of the reflection coefficient series is, of course, the objective of a spiking deconvolution process; now we ask also that all the reflection coefficient *magnitudes* should be correct. We can see immediately that we are unlikely to be successful (since, for one thing, our objective would require the complete removal of multiple reflections), and we can see also that use of it to obtain densities is subject to cumulative errors. However, it remains an objective.

3. *Absorption*

We have seen that spherical divergence, while it acts to diminish seismic amplitudes at distance, does not involve any loss of seismic energy—merely a spreading of it over a greater area of wavefront. Again, the processes of reflection and transmission at interfaces do not involve any loss of energy—merely a redistribution of it in the forward and backward directions. Absorption, however, is different; it diminishes seismic amplitudes, as a function of the distance travelled, by an irreversible conversion into heat.

This loss is known to be frequency-selective. A seismic pulse, representing a spectrum of frequencies, loses amplitude by a progressively greater absorption of its higher frequencies. In this sense, we note, the decay of amplitude introduced by absorption cannot properly be divorced from the change of spectrum.

Nowadays we accept that absorption in dry earth materials is related to a power of frequency very close to the first. This can be made eminently reasonable if we consider the seismic wave emanating from a sinusoidal source into a large homogeneous expanse of rock material. If we "freeze" the pattern of particle displacements at a certain instant, we see a succession of alternate compressions and rarefactions. The distance between successive compressions is a wavelength, at the frequency of the source and the velocity of the material. Then, because of absorption, we see a decay in the pressure amplitude from one compression to the next. So if we accept an absorption coefficient proportional to the first power of frequency, we accept, substantially, that this decay in acoustic pressure over each wavelength is a constant (which we might expect to be characteristic of the rock in its given environment). The proportional loss, over one wavelength, is substantially independent of frequency; it is therefore usually expressed in decibels per wavelength.

In deference to the theoretical workers, we should pause a moment to note that this simplified view ignores certain mathematical difficulties; nevertheless it seems to be sufficiently close to reality to warrant our using it for the present.

Let us illustrate the implications for a rock material having an absorption characteristic of 0.2 dB/wavelength and a velocity of 3 000 m/s, and let us consider a path length of 300 m in this material. At a frequency of 100 Hz this distance represents ten wavelengths. We expect each of the ten compressions to be 0.2 dB less in amplitude than the one before; thus the amplitude of the second compression is about 98% of that of the first, the third 98% of the second, and so on. At any other frequency the decay is likewise exponential. So, over the 300 m distance (corresponding to ten wavelengths at 100 Hz, or to one at 10 Hz) the loss is 0.2 dB at 10 Hz, 1 dB at 50 Hz, and 2dB at 100 Hz. Thus it is a simple matter to draw the effect of absorption on the spectrum of the propagating pulse; this is done in figure 4.

So we accept that decay of any sinusoidal component is exponential (in any one material), but what can we say about the amplitude of the composite pulse? Alas, very little; it all depends on the characteristics of the pulse which constituted the "input" to the absorbing earth—on the characteristics of the seismic source.

Under such circumstances our usual approach is to see what would happen to a pure spike input, and to reason onwards from that. So, adopting a spike input, we find that there are immediately two approaches we must consider. The first is appropriate to any circumstances in which we can fairly accept that we observe a single pulse in isolation (so that we can actually measure the amplitude of a selected peak of the pulse); the second applies when we see only a complex of overlapping pulses.

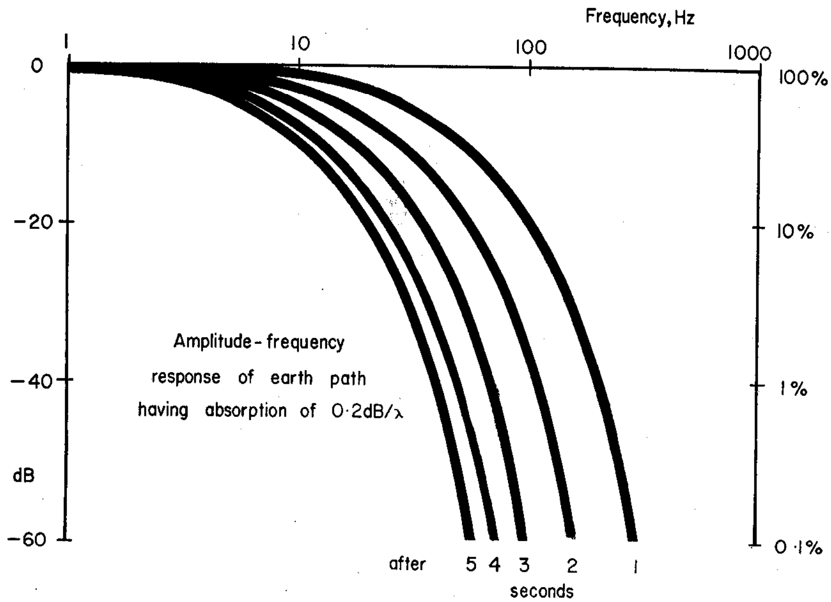


Fig. 4. The progressive high-frequency loss with increasing travel-time, illustrated for a uniform absorption of 0.2 dB/wavelength.

For a single pulse in isolation, two features affect the peak amplitude of the pulse: the peak amplitude decays as the higher frequencies are absorbed, and the peak amplitude decays as the pulse is lengthened by dispersion. The second effect occurs because the velocity of propagation of the sinusoidal components is slightly dependent on frequency, so that components which are in phase at the peak of the pulse early in its history are no longer exactly in phase at later times. To quantify the effect of dispersion on the peak amplitude of the pulse we must make some assumption about its magnitude; the assumption which is both convenient and physically reasonable is the minimum-phase assumption. (For a rudimentary account of minimum-phase behaviour, dispersion, absorption and other matters related to the present discussion, see section 2.3.12 and chapter 3.1 of volume 1 of Evenden, Stone and Anstey, 1970. For a more advanced account, including some practical evidence, see O'Brien, 1969). On this basis, we can display the pulses which correspond to the amplitude spectra of figure 4, and see the effect of absorption on their peak amplitudes; this is done in figure 5.

As we can see, the decay is very rapid at early times; the flat spectrum of the spike means that a considerable proportion of the spike energy is carried at the high frequencies, and this is quickly lost.

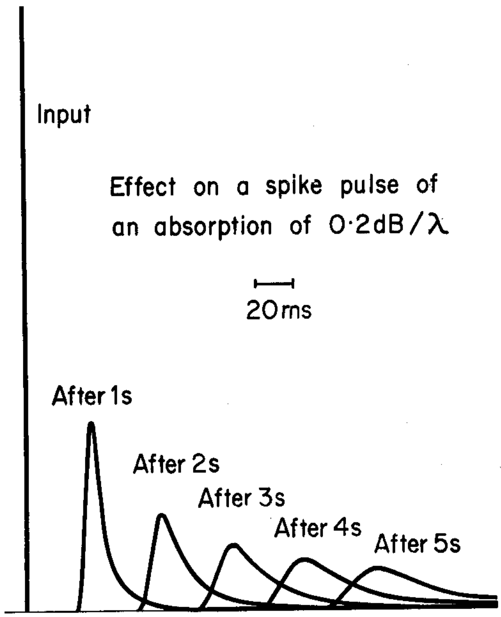


Fig. 5. The absorptive effect of fig. 4 translated into the time domain, on the assumption of minimum phase.

So we must accept the relevance of the source characteristics—if the source does not emit high frequencies, this rapid collapse of amplitude does not occur. Thus when we said earlier that in some practical cases spherical divergence accounts for the majority of the observed decay, we can guess that this indicates a *low-frequency narrow-band source*. There is nothing we can say about the decay of peak amplitude with time, until we know the characteristics of the source pulse (Gurvich and Yanovskii, 1968).

When we are not concerned with a specific pulse observed in isolation, but with a complex of overlapping pulses, one aspect of the problem changes. Manifestly, the broadening of the pulse by absorption and dispersion must increase the chances of overlap; manifestly, also, the resultant amplitudes may be increased or decreased by the overlap, according to the reflector signs, the reflector spacing and the pulse shape. To obtain a useful generalization we must go all the way to a reflection coefficient series having close but random reflector spacing (so that all reflected pulses overlap several or many times), and then look at *average* conditions within a window. Under these circumstances, clearly, dispersion loses its significance; it yields a broadening which is additional to that produced by absorption, but which, unlike the latter, does not involve a loss of energy. Looking at a window containing a random superimposition of pulses, we see dispersion merely as a phase effect which modifies the shape of the composite waveform without changing the energy evident in the window.

We have said that a knowledge of the source characteristics is necessary before we can calculate the amplitude decay due to absorption. The source characteristics, of course, are certain to be different from one source to another, and may also be different from one shot to another. Although this is really all that can be safely said, one would feel guilty about abdicating the discussion on such an unsatisfactory note. So let us consider at least one specimen case;

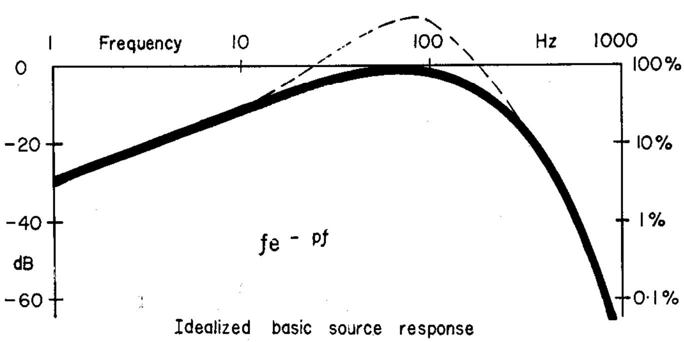


Fig. 6. The full line indicates a tractable basic form for the spectrum of the pulse generated by a small explosive charge in a competent material; normally a resonant peak (such as that shown dashed) is superimposed on this basic form.

for example, let us consider some reasonable form of source spectrum such as that illustrated by the heavy line in figure 6. Then we can show that this basic curve dictates an average amplitude decay, in the presence of absorption, which follows slightly less than a — 3/2 power of travel time.

In practice most sources exhibit a response more peaked than this (such as the one shown dashed in figure 6); obviously the effect of the peaked response superimposed on the basic curve is to modify the decay, to a greater or less extent.

We can summarize our preparatory discussion of absorption thus:

—It is reasonable to accept, at the present stage of our knowledge, that absorption varies very nearly with the first power of frequency.

—This means that the loss in decibels over a fixed distance in a single medium is proportional to frequency, or that at a single frequency the loss in decibels is proportional to travel time.

—The effect of this loss on the amplitude of the propagating pulse cannot be established until the source characteristics are known.

—As a generalization of the last item, the decay appearing as a result of any frequency-selective effect which is progressive with travel-time must depend also on the constant frequency-selective effects (source, detectors, instruments, filtering) along the path of the signal.

—It is probably reasonable to expect that the absorption mechanism exhibits minimum-phase behaviour.

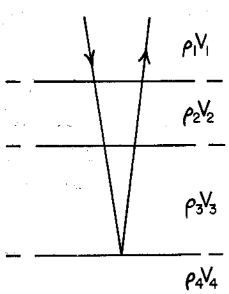
—The decay of amplitude of a single pulse observed in isolation is slightly greater than that of the average amplitude of a profusion of overlapping pulses.

—Only by chance could the pulse amplitude decay conform strictly to the popular exponential. (We note in passing that this does not exclude the use of exponential compensations for particular purposes. Before dereverberation, for example, we may be forced to use them; after dereverberation we may remove their effect and then apply a better correction if we know one.)

4. *Interface transmission losses*

Again invoking conservation of energy, we know that energy reflected from an interface is not available to be transmitted through it. Clearly, the larger the reflection coefficient, the greater is the transmission loss. We shall need to employ this again later, so let us take particular note of it: More up, less down.

The relationships between the reflection and transmission coefficients are depicted in figure 7. Clearly the transmission loss is unaffected by the sign of the reflection coefficient. Setting aside spherical divergence and absorption for the moment, we can see that the amplitude of a seismic reflection is the product of its own reflection coefficient with the product of all the two-way transmission coefficients of the interfaces above it.



Two-way Transmission Coefficient $= \frac{4\rho_1 V_1 \rho_2 V_2}{(\rho_2 V_2 + \rho_1 V_1)^2}$
(pressure amplitude, normal incidence)

$= 1 - R_{12}^2$

Amplitude of reflection from third interface $= R_{34}(1 - R_{12}^2)(1 - R_{23}^2)$

Fig. 7. The relationship between the transmission and reflection coefficients.

Are transmission losses a major effect, or a minor one? Intuition is not a very good guide on this question, and it helps to have before us some illustrative values. Figure 8 gives the two-way transmission loss as a function of the number of interfaces for reflection coefficients of $\pm$ 0.05, $\pm$ 0.1 and $\pm$ 0.2.

Our first conclusion is that the transmission loss associated with a single reflector—even a strong one—is virtually insignificant. We would expect a handsome reflection from a reflection coefficient of 0.2, and we might feel that there would be a major diminution of reflections below it; we see, however, that such reflections are diminished by only 0.4 dB, or 4%. The corresponding diminution introduced by a reflection coefficient of 0.05 is 0.02 dB (0.2%) which

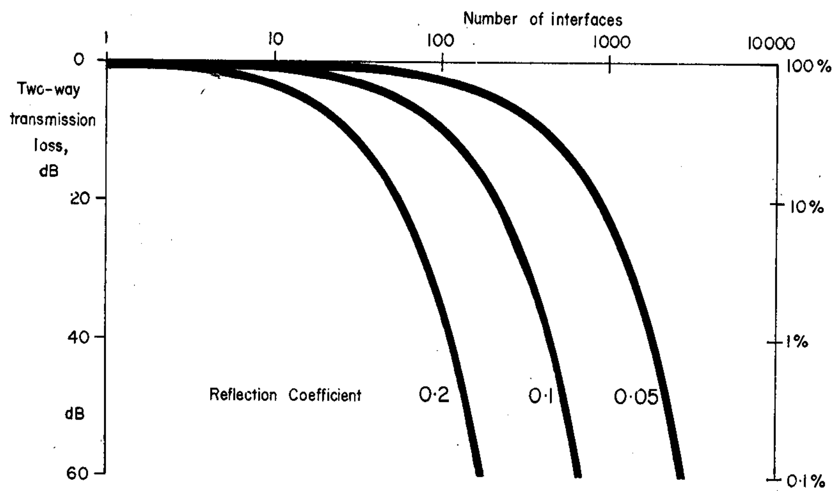


Fig. 8. The two-way transmission loss through a number of interfaces, for a range of reflection coefficients.

is in itself negligible. So if we think of the crust of the earth in terms of a few major layers (representing the coarsest division into geological epochs), we are led to expect quite small transmission losses.

The second conclusion, however, is that the cascaded transmission loss through a great number of interfaces is certainly not negligible. In particular, a large number of individually insignificant interfaces can have at least as great an effect as a few major ones. And, since we know from our first glance at almost any outcrop that the earth's stratification can be very fine, we realize that we have to start thinking rather carefully about geology before we can assess the true significance of transmission losses.

In particular, we find we have to make an immediate distinction between two extreme types of stratification, and the sedimentary processes which give rise to them. We illustrate these in figure 9, using artificial acoustic logs. Both logs show the same systematic increase of acoustic impedance with depth; however, the upper one implies a profusion of thin layers tending to alternate in their ρV values, while the lower one implies slow and progressive variations of ρV value. As geophysicists we would describe the upper log as high-frequency, the lower one as low-frequency. The more fundamental description, however, must be the geological one: we think of the upper log as representing thin layers laid down by a *cyclic pattern of sedimentation* which systematically tended to interleave high-velocity and low-velocity materials, while we think of the lower log as representing a *transitional pattern of sedimentation* which systematically tended to make steady gradations of velocity within basically thick layers.

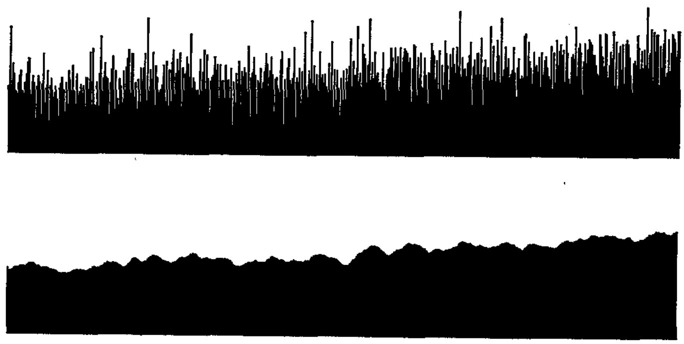


Fig. 9. Artificial acoustic logs prepared to illustrate the geophysical significance of cyclic layering (upper) and transitional layering (lower).

We shall use the terms *cyclic* and *transitional* to describe these two types of layering; in doing so, however, we note that our present concern is with the *acoustical* properties of the layering, and that cyclic sedimentation in this sense need not correspond exactly with cyclic sedimentation in the geological sense.

The relevance of the distinction between cyclic and transitional layering, in the present context, is brought home to us when we compare the transmission loss for the two cases of figure 9; for the lower log it is quite insignificant, while for the upper log it is more than a thousand times as great, and certainly significant.

This happens, of course, because the reflection coefficients in the lower case are smaller, so that the transmission loss is also smaller. But, within the constraints on velocity and density known to exist in the real earth, this is inevitable for a transitional log; large reflection coefficients and large transmission losses can be maintained only if the large reflection coefficients tend to alternate in sign.

Thus we see several clear situations. We see the situation of a massive layer which contains no significant reflectors and therefore contributes no significant transmission loss. We see the situation of a single interface having a large reflection coefficient and a minor transmission loss. We see the situation where this interface is transitional instead of abrupt; in this case the progressive transition makes the transmission loss even less significant—just as an acoustic horn matches between low and high impedance, and so reduces the loss. And we see the situation where thin layers tend to alternate between high and low ρV values, and so provide (if there are many of them) the possibility of large transmission losses.

So, in hopes that we may be led to a technique for distinguishing between transitional and cyclic geology, we ask: How can we assess the magnitude of transmission losses in the real earth?

This proves an unexpectedly knotty problem. It is indisputable that, over a given up-and-down path in the earth, there must be a definite and altogether real transmission loss. But the obvious way to obtain a measure of it—from a velocity log—proves to be full of difficulties.

First, we observe that a velocity log taken with a 1 m receiver spacing is much more active than one taken with a 2 m spacing. This reminds us that a velocity log does not identify (except in a blurred sense) layers having a thickness significantly less than the receiver spacing; the transmission loss computed from a cyclic log at 1 m spacing is greater than that obtained with a 2 m spacing. How far does the effect go? Geologically, we feel that, although very fine layering obviously exists, reflection coefficients between very thin layers are likely to be small. But the potential number of very thin layers is enormous.

Second, we know that not every wiggle on a velocity log represents a corresponding formational change; errors associated with the borehole are inevitable, and the difficulties of compensating these increase as the receiver spacing is reduced.

Third, if we attempt our transmission-loss evaluation digitally, we must be careful to ensure a proper sampling interval. Obviously much of the early implementation of synthetic seismograms was in violation of this; probably this did not matter too much for those applications, but it is essential in any attempt to evaluate transmission losses.

Fourth, we have to think about the nature of geological "interfaces". Sometimes, for sure, they are real discontinuities, properly represented by the simple equation; others may be very smooth gradations (caused, for example, by variations in porosity with progressive changes of grain size) within which the transmission loss is virtually zero.

Finally we have to ponder the acoustics of the situation. The cited equation for transmission loss applies to plane waves, and we usually side-step this by restricting ourselves to the far field, where our spherical waves are almost plane.

But if we are concerned with very thin layers this may not be defensible (see, for example, Hagedoorn, 1954).

So we have to admit that, although the concept of a transmission loss in the real earth is a clear one, we are not well placed to assess the magnitude of the loss.

If we do the best we can, with the logs available today, we emerge with what appears to be a ridiculous result. As we have said earlier, a record from a narrow-band low-frequency source, properly compensated for spherical divergence, shows very little decay attributable to transmission loss—at most a few decibels per second. But values of transmission loss computed from velocity logs often work out at 40-50 dB/s—a figure which, if real, would mean that the seismic reflection method could not possibly work as it does.

So something is wrong. Probably part of the reconciliation lies in the considerations set out above, but part—the greater part—must involve multiple reflections. Just as it is physically ridiculous to think of reflection without multiple reflection, so it is improper to consider transmission without multiple reflection.

5. *Multiple reflection effects*

For many of us, the first intimation that multiple reflections affect the amplitude of "primary" reflections came with the introduction of digitally-generated synthetic seismograms. In those days it was customary to calculate at least three synthetic traces: primaries without transmission losses, primaries with transmission losses, and primaries with all multiples and with transmission losses. Just as we have seen above, the second of these—primaries with transmission losses—usually decayed away to nothing so quickly as to be useless. The most obvious effect of including the multiples was to increase the amplitude of the *primaries*, sufficiently to offset most of the transmission losses.

The explanation—that primary paths are systematically reinforced by very-short-delay "peg-leg" multiple paths—was given and developed by Anstey (1960), Trorey (1962), d'Erceville and Kunetz (1963), Bois and Hemon (1963), Bois, Hemon and Mareschal (1965), Delas and Tariel (1965), Mikhailova, Pariiskii and Saks (1966), and Berzon (1967).

In retrospect, we can see that the explanation was always implicit in the classical acoustic exercise, "the case of the thin plate." In figure 10*a* we see the basic situation: the direct transmitted signal is followed after a short delay by a 2-bounce multiple reflection whose amplitude, referred to the direct transmitted signal, is just the product of the upper and lower reflection coefficients. The most important feature of this situation is that the sign of the multiple reflection is *always the same as that of the direct signal*; the reflection coefficients are opposite in sign when viewed from above, and one of them is in fact viewed from below—so the product is always positive.

Our first reaction is to see whether we are looking at a major effect or not. Quickly we note that if both reflection coefficients have a magnitude even as high as $\frac{1}{2}$, the multiple reflection has an amplitude of only $\frac{1}{4}$ of that of the direct signal. For more realistic magnitudes, the multiple is very small. However, before we discard the effect as insignificant, we should consider the case of Figure 10*b*, where we postulate that, somewhere along its path, the direct signal

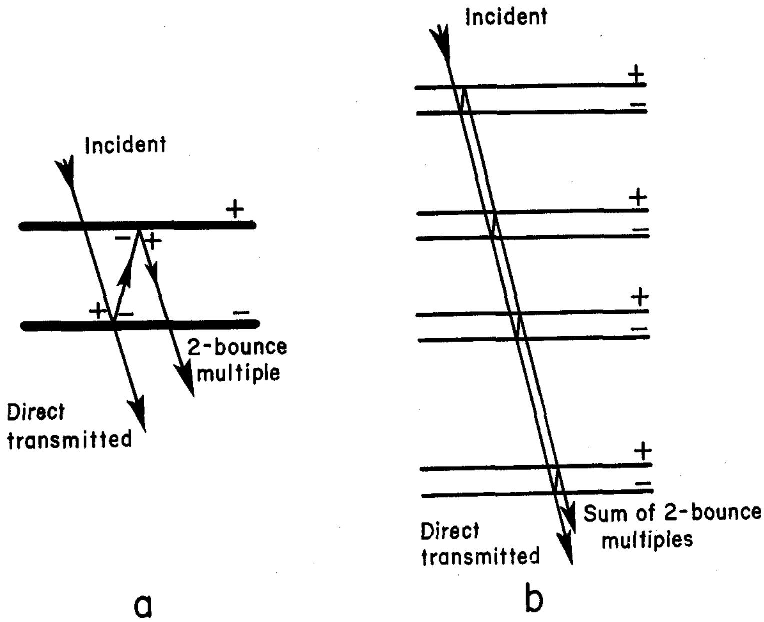


Fig. 10. (*a*) The basic thin plate, defined between interfaces having reflection coefficients of opposite sign.

(*b*) The cumulative effect of the multiple reflections from four such thin plates.

encounters four such thin plates. Then if the reflection coefficients are again $\frac{1}{2}$, we can see that the composite multiple reflection is now *equal to the direct signal*. When we include the return path through the same sequence, the composite multiple reflection has double the amplitude of the direct signal.

If the reflection coefficients are of magnitude 0.1, then it takes 50 thin plates before the multiply-reflected signal becomes equal to the two-way direct signal, but the conclusion is eventually the same: *the multiply-reflected signal in a series of thin plates bounded by interfaces of opposite polarity is always of the same sign as the direct transmitted signal, and tends to overtake it in amplitude*. At this stage we can see, in a gross sense, that in a cyclic sequence of layering—a succession of thin plates—there is an inbuilt multiple-reflection mechanism which acts to compensate the large transmission loss that would otherwise occur. Clearly, we are now concerned to determine the *degree* of this compensation: will it be so great that we lose all hope of distinguishing between cyclic and transitional layering by their effect on amplitudes?

Basically, this is a matter of geology; so, let us return for a moment to the geological matters broached in the last section. In particular, let us return to the question: Do geological considerations lead us to expect any connection between the thickness of a layer and the reflection coefficients at its boundaries?

The thick layers, we have said, often represent geological epochs. In the nature of things, there can be only a few of them. The interfaces bounding them may well have large reflection coefficients; the chances are that both are positive.

At the other end of the scale, we have guessed that the very thin layers are likely to be bounded by very small reflection coefficients—we can scarcely conceive large reflection coefficients separated by a matter of centimetres. If the reflection coefficients are small, we can allow the possibility of sequences of transitional layering, while still keeping the overall variations of acoustic impedance within observed limits. We as geophysicists cannot say whether transitional or cyclic layering is the more likely; that is a question for the geologists.

Between the very thin and the very thick layers there must be some range of thicknesses where appreciable reflection coefficients first become possible. Over and beyond this range, the observed limits on acoustic impedance mean that *sustained cyclic layering is more likely than sustained transitional layering.*

So these considerations lead us to expect a middle range of layer thicknesses, bounded by significant reflection coefficients tending to be opposite in sign.

If these guesses have any foundation, we are immediately at odds with one of the favourite assumptions in the theory of seismic processing. Often, in processing, we invoke the assumption that the reflection coefficient series is a train of spikes of random amplitude, spacing and polarity; now we are saying that the *earth's stratification is the result of natural laws, that these provide some predictable constraints, and consequently that the outcome is not completely random.*

Is there some simple check we can make, to resolve the issue? Yes, there is; the assumption of randomness usually expresses itself as an assumption that the auto-correlation function of the reflection coefficient series is a simple spike (so that the auto-correlation function of the ideal seismic record is the auto-correlation function of the seismic pulse)—we can check whether this is true.

Let us try it first on the two synthetic examples of figure 9. Figure 11*b* is the auto-correlation function of the reflection coefficient series corresponding to the cyclic log of figure 9*a*; figure 11*a* is that corresponding to the transitional log of figure 9*b*. Clearly *neither* is a simple spike. In particular, we note that the first few values of the transitional auto-correlation function are all positive, whereas the second value of the cyclic auto-correlation function swings strongly negative. We shall see the significance of this in a moment.

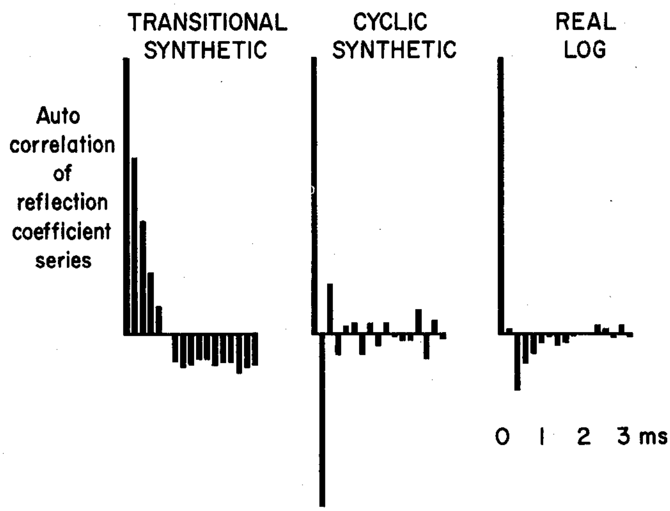


Fig. 11. The auto-correlation functions corresponding to (left) an artificial transitional log, (centre) an artificial cyclic log, and (right) the real log of fig. 12.

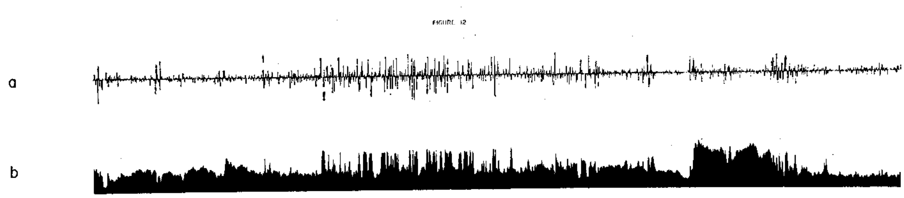


Fig. 12. The reflection coefficient series (*a*) corresponding to a segment of a real acoustic log (*b*).

In figure 11*c* we see the auto-correlation function of a "real" reflection coefficient series. In fact, the reflection coefficient series is that shown in figure 12*a*, and the real velocity log from which it was derived is shown in figure 12*b*. The derivation was made on the usual assumption that density variations can be neglected; any case we can demonstrate under this restriction is likely to remain essentially sound when the restriction is removed. The log represents about 1400 m of depth, and 0.343 s of one-way time. The original sampling, before conversion to time, was at 0.765 m of depth, chosen to be less than the receiver spacing of the logging tool. To the eye, the reflection coefficient series could well be random.

But the auto-correlation function (figure 11c) says no. We can see the spike at zero lag, of course, and the values for lags beyond about 2ms are compatible with randomness, but between zero and 1 or 2 ms the values have a different message. They are telling us that, for layer thicknesses up to about 10 m, the stratification shows systematic deviations from randomness.

If we recall the actual operations used in the construction of the auto-correlation function—shifting, multiplying, and adding—we can see very easily that the auto-correlation function of a reflection coefficient series has a direct physical significance. The value at the first lag represents the sum total of all the 2-bounce multiple reflections occurring in layers of one unit of thickness. The value at the second lag represents the sum total of all the 2-bounce multiple reflections occurring in layers of two units of thickness. Similarly, values at higher lags represent 2-bounce multiples of longer periods. If the value is positive, it means that the sum total of all the corresponding 2-bounce multiples is of opposite polarity to the direct transmitted signal; if it is negative, the multiples reinforce the direct signal.

So the auto-correlation in figure 11*c* is confirming our geological guesses about the likely relation between the thickness of a layer and the magnitude and signs of the interfaces bounding it.

It is true that the equivalence is not exact; our earlier thinking regarded a layer in the sense of a geological entity, whereas figure 11*c* regards a layer as defined by *any* pair of interfaces. Further, the physical interpretation of the auto-correlation function in terms of 2-bounce multiples ignores the transmission loss in all intervening interfaces. Nevertheless the auto-correlation function suggests two conclusions for the particular log under study:

—Layers of very small thickness (typically about 1 m or less) show a weak tendency to be of transitional type, being bounded by interfaces of like sign. The small magnitude of the first lag value could be due either to a fortuitous offsetting of transitional and cyclic effects, or—and this seems more likely—it could be confirming our guess that very thin layers are likely to be bounded by small reflection coefficients.

—Layers in the thickness range 1-10 m tend to represent cyclic changes, being bounded by interfaces of opposite sign. This is a clear and positive effect.

So the evidence, at this stage, allows us to say that the earth may contain:

—A very large number of very thin layers, whose boundaries have small reflection coefficients and introduce small transmission losses; these losses may be increased somewhat (if the layers are transitional) by multiple reflection effects.

—A smaller (but still large) number of less thin layers, whose boundaries tend to have appreciable reflection coefficients but to be of opposite sign; the very large transmission losses to be expected in this case tend to be offset by multiple reflection effects.

—A small number of thicker or much thicker layers, whose boundaries tend to have large reflection coefficients; these interfaces give rise to the reflections we see on normal records, but their comparatively small number means that the transmission losses introduced by them are minor.

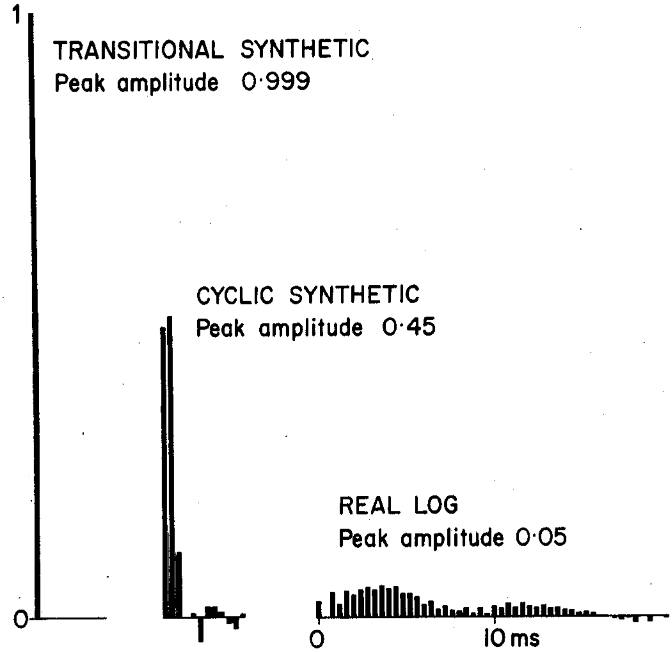


Fig. 13. The transmission response (that is, the impulse response of the two-way transmission path) for (left) the artificial transitional log, (centre) the artificial cyclic log, and (right) the real log of fig. 12.

Before we leave the auto-correlation function, we should note this interesting fact: We can manipulate the positive-lag auto-correlation function to approximate the actual time form of the 2-bounce multiple reflections by removing the zero-lag value, by reversing the sign of all other values, and by doubling the time scale to yield two-way multiple reflection times.

Although this gives us an easy way of assessing whether a particular reflection coefficient series will systematically reduce its transmission loss by its 2-bounce multiple reflections, it leaves us wondering what is the effect of the 4-bounce and higher-order multiples. For this, and to eliminate the approximation involved in neglecting transmission loss within the multiple part of the path, we must go through a complete ray-tracing process to find the form of the complete transmitted signal. It makes good sense to do this for two-way transmission; then we say that we will inject a spike pulse at the top of the layered sequence represented by the log, and calculate the form of a pure isolated reflection as it would be after two-way transmission down and back. We do this in figure 13*a* for the synthetic cyclic log of figure 9*a*, in figure 13*b* for the synthetic transitional log of figure 9*b*, and in figure 13*c* for the real log of figure 12. (The techniques for this type of calculation have been given by several previous workers; see, for example, Baranov and Kunetz, 1960; Trorey, 1962.)

As we expect, the spike pulse is scarcely changed, either in amplitude or in form, by transmission through the transitional sequence (figure 13*a*). In fact there is a small tail added (Bortfeld, 1960), but for a log of the limited extent involved here it is too small to be significant. The change of amplitude due to transmission loss is from 1 to 0.999.

The cyclic sequence, however, produces a much more marked effect (figure 13*b*). The transmission loss (that is, the diminution of the first point of the transmitted signal) is from 1 to 0.44. A significant positive tail is added, extending to three points. The sum of the amplitude values for the first three points is 0.994. This, obviously, is very interesting; it is telling us that the decrease of amplitude caused by transmission is at least partially compensated by multiple reflection—at the expense of a smearing-out over time. We begin to sense that there will be great difficulty in distinguishing between transitional and cyclic layering by studies on amplitudes alone.

Our greatest concern, of course, attaches to the real log, for which the two-way transmitted pulse is given in figure 13*c*. The change of amplitude due to transmission loss is from 1 to 0.027, which obviously cannot be the effective value. A very significant positive tail, extending to some 16 ms, is added by the multiple reflections; the sum of the amplitude values over this systematically reinforcing tail is 0.874. Clearly, the effect of the very-short-delay multiples dominates the directly transmitted signal.

And this, we remember, is with a two-way travel time of only 0.686 seconds. If we visualize a deep earth section having the same layering characteristics as are evident in our short piece of log, we can auto-convolve figure 13*c* sufficient times to represent the two-way travel path to any desired depth. Figure 14*a* is a repeat of figure 13*c*; figure 14*b* represents the approximate form of the output after a transmission corresponding to two-way travel time of 1.372 s, figure 14*c* that corresponding to 2.744 s, and figure 14*d* that corresponding to 5.488 s.

Of the several conclusions implicit in these diagrams, let us first stress the one concerned with the very first point—the direct arrival—which obviously becomes quite negligible in all of them.

At reflection times of usual interest, it would make no significant difference if the direct "primary" reflection path did not exist; the useful seismic information is carried by the very-short-delay multiple reflections.

Now let us look at the *form* of the transmission responses of figure 14. Clearly one effect of the lengthening path is to broaden the output pulse; in a

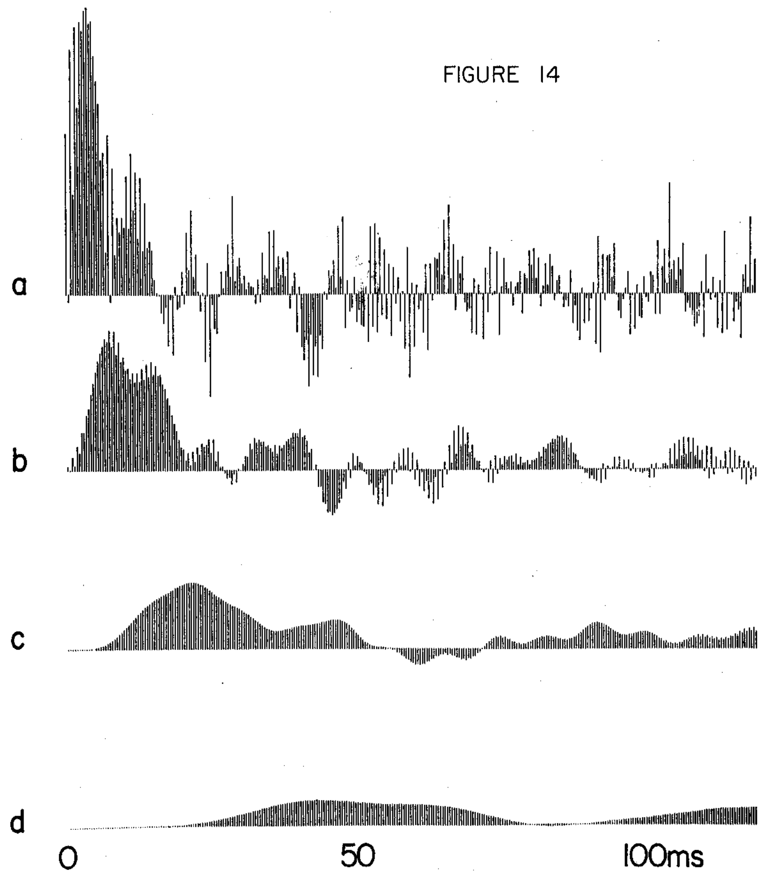


Fig. 14. Illustration (*a*) repeats and extends the two-way transmission response of the real log in fig. 13. Illustrations (*b*), (*c*) and (*d*) are successive auto-convolutions of (*a*), representing the effect of longer path lengths in a statistically similar sequence of layers.

coarse sense, the broadening is similar to that produced by absorption (although the mechanism is entirely different). And, just as the broadening produced by absorption is associated with a high-frequency cut, so the broadening produced by very-short-delay multiples implies a high-frequency cut.

In fact, this conclusion was always present in our simple argument "More up, less down". For one look at the reflection coefficient series of figure 12*a* tells us that the total signal reflected back to the surface must have a low-frequency cut; the high-frequency appearance—the cyclic nature of the sedimentation—virtually guarantees this. Then we can apply our simple argument to spectra just as convincingly as to amplitudes, and conclude that if the reflected signal has a low-frequency cut the transmitted signal must have a high-frequency cut.

Of course, we should be able to do better than just "More up, less down". In the Appendix is set out the derivation of an approximate relationship between

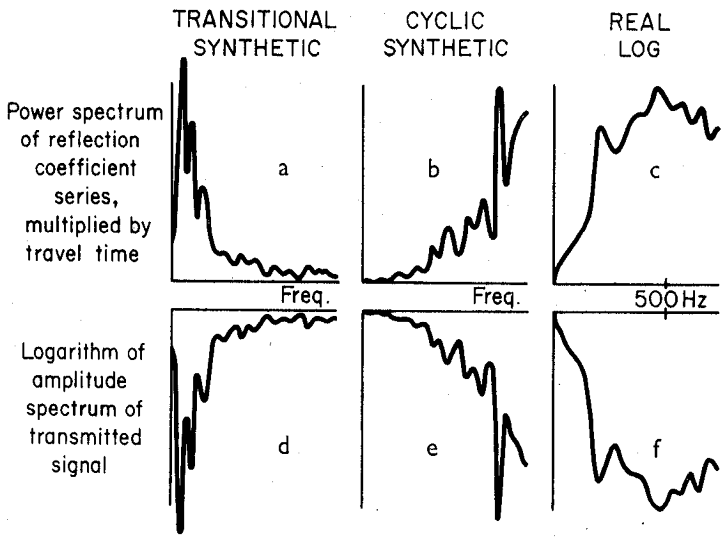


Fig. 15. The anti-correlation between the spectra of the reflecting sequence and of the signal transmitted through it. Frequencies which are selectively reflected are poorly transmitted. More up, less down.

the amplitude spectrum $T(\omega)$ of the transmitted pulse and the power spectrum $R(\omega)$ of the reflection coefficient series:

$$T(\omega) = e^{-R(\omega)t}.$$

This relationship is simply checked by comparing the power spectrum of a given length of the reflection coefficient series with the logarithm of the amplitude spectrum of the pulse transmitted through it. In figure 15 we do this for our two synthetic logs and for the real log. The reflection coefficient series for the illustrative transitional log has the expected low-frequency spectrum (figure 15*a*), and the signal transmitted through it has a correspondingly inverse spectrum (figure 15*d*). The reflection coefficient series for the illustrative cyclic log has the expected low-frequency cut (figure 15*b*), while the signal transmitted through it has an inverse high-frequency cut (figure 15*e*).

Both examples fit well with the approximate relation above. Particularly satisfying is the real log itself; the power spectrum of the reflection coefficient series (figure 15*c*) is accurately mirrored by the amplitude spectrum of the transmitted pulse (figure 15*f*).

This excursion into the frequency domain gives us an alternative and interesting way of looking at the combined effects of transmission loss and very-short-delay multiple reflections. For if we imagine a reflection coefficient series whose spectral structure is such as to have no content at a particular frequency, then there is, in effect, no overall loss at that frequency. However, since the transmitted signal is minimum-phase (and since, therefore, the phase at that frequency depends on the phase at all other frequencies), that frequency can experience a delay even though there is no loss (d'Erceville and Kunetz, 1963; Sherwood and Trorey, 1965).

Further, just as we were beginning to lose all hope of distinguishing between transitional and cyclic sequences by their effect on amplitudes, we see now that there may be additional help available in the frequency domain; specifically, the pulse transmitted through a transitional sequence has a low-frequency cut, while that transmitted through a cyclic sequence has a high-frequency cut.

The formidable difficulty in deriving benefit from this, of course, is that of distinguishing between the effect of the high-frequency cut due to cyclic layering and that due to absorption. Both are progressive, both involve a loss of amplitude and a broadening of the transmitted signal. And, fortuitously, the degree of high-frequency cut associated with cyclic layering may look very much like a constant dB/wavelength effect (at least over a restricted frequency band). Indeed, if we draw a smooth curve to approximate the spectrum of figure 15*f* over the first 100 Hz, we emerge with a high-frequency cut of about 0.3 dB/wavelength.

The magnitude of this figure immediately throws all our thoughts into disarray. For it raises the clear possibility that the loss of high frequencies due to passage through a cyclic sequence of layers may be greater than the loss of high frequencies due to absorption—the multiple-reflection effect may dominate the absorption effect. This, in turn, says that our records might look much the same if absorption did not exist. Further, in turn, we are led to question the magnitudes that have been quoted for absorption—is it possible that the experimenters have been ascribing to one mechanism an effect which actually owes much to another?

We do not know. What seems most likely is that the two effects co-exist, both contributing a high-frequency cut and one sometimes dominating the other. The seismic pulse returned from any discrete reflector is therefore the interaction or convolution of the pulse shape contributed by the source, the pulse shape contributed by absorption, and the pulse shape contributed by the very-short-delay multiple reflections.

In figure 16 we see along the top line three possible breadths for the pulse representing the combined effects of absorption and the source. In the second line we see these reproduced at 0.027 of the amplitude, to illustrate what would be the loss in amplitude caused by two-way transmission through the log of figure 12 in the (unreal) absence of the very-short-delay multiple reflections. In the third line we see the pulses of the first line convolved with the two-way transmission response of figure 14*a*, incorporating the very-short-delay multiple reflections. This convolution provides the means for recombining, in effect, the amplitude contributions smeared out over time by the multiple reflection process. The amplitude actually obtained depends, clearly, on the relative breadths of the absorption pulse and the transmission pulse, and on the nature of the high-frequency and low-frequency pulse-shaping effects near

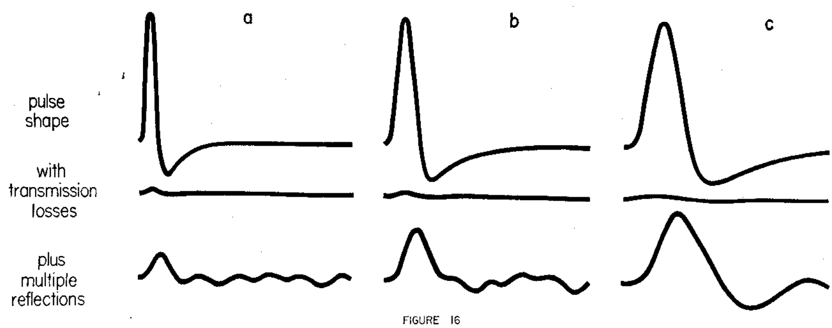


Fig. 16. The effect of convolving three seismic pulse shapes with the pulse formed by transmission through the log of fig. 12, without and with the effect of the very-short-delay multiple reflections.

the source. In general terms, however, we can see that pulses of likely shapes can be transmitted through a strongly stratified earth without amplitude losses of more than a few decibels per second—the observed values.

Summary and Conclusions

The loss of amplitude associated with the geometrical divergence of a wavefront is physically a clear and simple effect. It can be compensated with reasonable safety and reasonable accuracy. When this is done, little decay remains on records using low-frequency narrow-band sources.

After correction for divergence, the amplitude of a reflection depends on its own reflection coefficient and on all the losses incurred above it. If the reflection is discrete the process of reflection itself is not frequency-selective (though, of course, "tuning" effects occur if the reflector is part of a complex). The losses occurring above the reflector are all frequency-selective; the main ones are absorption and the combined effects of transmission and multiple reflection at interfaces. Before we can achieve the desired measurement of reflection coefficient we must be able to quantify these losses; this must require the study of the amplitude loss and the spectral change in combination. The explosives manufacturers can draw some consolation from the observation that these effects are best measured on records from a wide-band source.

The expected effect of absorption can be presented fairly simply; however, the mechanism of absorption—the cause rather than the effect—is not very clear. Further, a case can be made that the magnitude of the effect is in question, since some of the early experimental work may not have taken sufficient account of very-short-delay multiple reflections.

The effects of very-short-delay multiple reflections on "primary" reflection amplitudes and spectra are critically dependent on the nature of the stratigraphy. The distinction between transitional and cyclic sedimentation seems to be a basic and a helpful one, though much more work is required to formulate the connection between the mechanism of sedimentation and the consequent constraints on the reflection coefficient series. This could prove an important and rewarding subject for academic research; although the synthetic seismogram is no longer fashionable, its usefulness is certainly not exhausted.

Transitional sequences, if they are to have realistic upper and lower bounds of acoustic impedance, can show only a small transmission loss; this loss is increased slightly by the effect of very-short-delay multiple reflections. The multiple-reflection effect for a transitional sequence has a low-frequency cut.

Cyclic sequences, still within the same bounds, can have enormous transmission losses; these losses are largely offset, at low frequencies, by the effect of very-short-delay multiple reflections. The transmission process for a cyclic sequence therefore appears to have a high-frequency cut.

The log tested in the present work showed predominantly cyclic stratification in the range of layer thicknesses from 1 to 10 m.

There is an anti-correlation between the power spectrum $R(\omega)$ of the reflection coefficient series and the amplitude spectrum $T(\omega)$ of the pulse transmitted through it; the simple relationship $T(\omega) = e^{-R(\omega)t}$ seems to be a satisfactory approximation.

All of this amounts to a hope—but not a promise—that the combined study of amplitude decay and spectral change on seismic records can lead to some definition of the statistics of the reflection coefficient series, and that this in turn will be interpretable in terms of the type of geological sedimentation.

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APPENDIX

The transmission response of a set of thin layers

Consider a section whose acoustic response for normally incident energy can be completely described by a set of N reflection coefficients $r(j)$ equally spaced in time. We shall try to show that a frequency-domain relationship exists between the series r and its transmission response T.

Strictly, we deal with the early part of T only. We assume that the effective length of the transmitted pulse is sufficiently short that the differential transmission loss of its components can be neglected. This is not unrealistic for sedimentary layers. However, we do need a more restrictive assumption. If we define

$$a(l) = \sum_{j=1}^{N} r(j)\, r(j+l), \tag{1}$$

where l represents the delay of a multiple relative to the direct arrival, the expected value of $r(j)\, r(j+l)$ is taken as $a(l)/N$ (that is, the series is assumed stationary). Values of $r(j)$ outside the section are to be read as zeros for the purpose of expressions such as (1).

An impulse in layer N gives rise to a response in layer 0 which consists of a direct arrival and a set of multiply-reflected trains ${}_k s$ characterized by $2k$ internal reflections. If the direct arrival has unit amplitude, the first multiple is

$$ {}_1 s(l) = -\sum_{j=1}^{N} r(j)\, r(j+l), \qquad l > 0 \tag{2}$$

where only the differential transmission loss factor, of the form $\prod_{w=j+1}^{l-1} (1 - r_w^2)$, has been neglected. It is convenient to consider this as equivalent to a more general function $m(l)$ defined by

$$\begin{aligned} m(l) &= -a(l) \qquad l > 0 \\ m(l) &= 0 \qquad\quad\ \ l \leqslant 0. \end{aligned} \tag{3}$$

To evaluate the second multiple, we consider the expression

$$ c(l_1, l_2) = \sum_{j_1=1}^{N} r(j_1)\, r(j_1 + l_1) \sum_{j_2=1}^{j_1+l_1-1} r(j_2)\, r(j_2 + l_2) \tag{4}$$

which represents a contribution to ${}_2 s$ at a delay of $l_1 + l_2$. The suffixes on j and l now indicate the order in which the multiple reflections occur. We can approximate equation (4) by

$$\begin{aligned} c(l_1, l_2) &\simeq \sum_{j_1=1}^{N} r(j_1)\, r(j_1 + l_1) \sum_{j_2=1}^{j_1} m(l_2)/N \\ &\simeq \sum_{j_1=1}^{N} \{m(l_1)/N\}\{j_1\, m(l_2)/N\} \\ &\simeq \tfrac{1}{2}\, m(l_1)\, m(l_2). \end{aligned} \tag{5}$$

For an estimate of the total arrival on ${}_2 s$ at a delay l we sum this, writing $l - l_1$ for l_2:

$$ {}_2 s(l) = \sum_{l_1=1}^{l-1} \tfrac{1}{2}\, m(l - l_1)\, m(l_1). \tag{6}$$

This is a convolution and states that, if the multiples are normalized by the direct arrival, the second multiple is half the autoconvolution of the first; physically this makes sense if we think of m as the basic multiple-generating filter. The average amplitude of the first multiple within the section is half its final value; we would expect it to grow linearly. If we repeat this reasoning for the higher-order multiples it is found that a simple recursive relationship exists which can be written

$$ {}_k s = 1/k \; {}_{k-1} s * m \tag{7}$$

We can sum all these series in the frequency domain, defining

$$M(\omega) = \sum_{l=1}^{N} m(l)\, e^{-i\omega l\tau} \tag{8}$$

where τ is the two-way transmission time within a layer. The transform of the pulse is now given by

$$T'(\omega) = 1 + M(\omega) + \frac{M^2(\omega)}{2!} + \frac{M^3(\omega)}{3!} \ldots = e^{M(\omega)}. \tag{9}$$

For the seismic problem we are interested in two-way transmission. Happily we do not have to go through the argument again, as the direction of propagation affects only the way we number the reflection coefficients. The shape of the two-way transmitted pulse is obtained by squaring T':

$$T''(\omega) = e^{2M(\omega)}. \tag{10}$$

This still describes a pulse whose first arrival is unity. If the original impulse is unity, we have to multiply T'' by $\prod_{j=1}^{N} \{1 - r(j)^2\}$. We can approximate this by considering the relationship

$$\operatorname*{Limit}_{x\to\infty} \left(1 - \frac{v}{x}\right)^x = e^{-v}, \tag{11}$$

so that, for large N, we would expect $e^{-a(0)}$ to be a satisfactory estimate of the direct arrival. This gives our final pulse spectrum

$$T(\omega) = e^{-a(0) + 2M(\omega)}. \tag{12}$$

To derive useful information from this expression we could note that the amplitude spectrum of T is defined by the real part of the exponent, which is identical to the transform of $-a$. If we define the power spectrum $R(\omega)$ of the reflection coefficient series as the transform of a normalized by the travel-time $t = N\tau$, we can write this as

$$|T(\omega)| = e^{-R(\omega)t}. \tag{13}$$

Equation (12) defines T as a minimum-phase function, so that it has at least one property in common with the exact response (Sherwood and Trorey 1965).

As to the interpretation of the layer thickness τ, any analysis that uses the concept of a reflection coefficient is necessarily discrete, but it is reassuring to note that the transform of a reflection coefficient series can have an interpretation in terms of acoustic impedance q, since

$$\sum_{j=1}^{t/\tau} r(j)\, e^{-i\omega j\tau}$$

converges to

$$\int_0^t \frac{d(\log q)}{2\,du} \; e^{-i\omega u} \, du.$$

(Therefore the filtering of a q log is a legitimate procedure, but the filtering of log q is better.)

These considerations lead us to suppose that the power spectrum of a reflection coefficient series is a meaningful concept, which does not depend on the choice of τ.